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Selection Rules for Nonrenormalizable Couplings in Superstring Theories

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Abstract

We study nonrenormalizable coupling terms in Z_N orbifold models. Nontrivial selection rules of couplings are provided and cannot be understood in terms of a simple symmetry of effective field theories. We also discuss phenomenological implications of theses selection rules for the quark mass matrices.

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1. Introduction

Superstring theory is the only known candidate for the unified theory of all the interactions including gravity. Much work has been devoted in order to study phenomenological aspects of effective field theories derived from superstring theory. Nonrenormalizable couplings lead to renormalizable coupling terms with suppression factors after symmetries are broken like an anomalous U(1) symmetry breaking [1, 2, 3]. Thus nonrenormalizable couplings have been discussed in order to derive some types of hierarchical structures, e.g., mass matrices of quarks and leptons [4, 5], the μ-term [6], large masses of right-handed neutrinos [7] and so on. In ref.[5] it is shown that one can obtain the realistic quark and lepton masses and mixing angles within the framework of a simple extension of the standard model by U(1), using nonrenormalizable couplings. Hence it is important to investigate selection rules of nonrenormalizable couplings as well as renormalizable couplings in superstring theory.

Coupling terms are restricted by several symmetries of superstring theory. For orbifold models [8], selection rules of Yukawa couplings are discussed in refs.[9-13] and further selection rules of nonrenormalizable couplings are discussed in refs.[14, 2, 15]. In ref.[15] a nontrivial selection rule of nonrenormalizable couplings is derived in the case where states sit at the same fixed point of the orbifold. One cannot understand this rule in terms of symmetries of effective fields theories. In this paper we study nonrenormalizable couplings in Z_N orbifold models to derive other nontrivial selection rules. We also discuss phenomenological implications of theses selection rules.

This paper is organized as follows. In section 2 we study selection rules of nonrenormalizable couplings due to symmetries of a six-dimensional compactified space and the H-momentum conservation. In section 3 we discuss phenomenological implications of these selection rules, e.g. for the quark

mass matrices. We also comment on the CP phase. The last section is devoted to conclusion.

2. Nonrenormalizable couplings in orbifold models

In orbifold models, string states consist of the bosonic strings on the four-dimensional space-time and a six-dimensional orbifold, their right-moving superpartners and left-moving gauge parts. The right-moving fermionic parts are bosonized and momenta of bosonized fields, H_t ($t = 1 \sim 5$), span an SO(10) lattice. A Z_N orbifold is obtained through a division of a six-dimensional space R^6 by a six-dimensional lattice and its automorphism θ. We denote eigenvalues of θ in a complex basis $(X_i, \overline{X}_i)$ ($i = 1 \sim 3$) as $\exp[2\pi i v^i]$. For simplicity, we restrict ourselves to the Z_N orbifolds which are obtained by products of three two-dimensional orbifolds, i.e. Z_3, Z_4, Z_6-I and Z_6-II orbifolds. Their twists θ are shown in the second column of Table 1. To preserve the world-sheet supersymmetry, the SO(10) lattice is also divided by the shift v^t, whose fourth and fifth elements correspond to the four-dimensional space-time and vanish.

There are two types of closed strings on the orbifolds. One is the untwisted string and the other is the twisted string. For the θ^k-twisted sector T_k, the string coordinate x_ν ($\nu = 1 \sim 6$) has the following boundary condition:

$$x_\nu(\sigma = 2\pi) = \theta^k x_\nu(\sigma = 0) + e_\nu, \tag{2.1}$$

where e_ν is a lattice vector. A zero-mode of the twisted string satisfy the same equation as (2.1) and is called a fixed point. The fixed point is denoted by the space group element (θ^k, e_ν). Note that fixed points of θ^k are not always fixed under θ. Therefore we have to take linear combinations of twisted states corresponding to fixed points f in order to provide eigenstates under θ. Suppose that σ_f is a θ^k-twist field [10] corresponding to f, which is fixed

under θ^m with the minimum number m. Eigenstates of θ are obtained by linear combinations as [11, 12]

$$\sigma_{f\gamma} \equiv \sigma_f + \gamma^{-1}\sigma_{\theta f} + \gamma^{-2}\sigma_{\theta^2 f} + \cdots + \gamma^{1-m}\sigma_{\theta^{m-1} f}, \tag{2.2}$$

where $\gamma = \exp[2\pi i n/m]$ and it is an eigenvalue of the Z_N twist. The θ^k-twisted sector has the H-momentum $\tilde{p}^t = p^t + kv^t$, where p^t is a quantized momentum on the SO(10) lattice. The SO(10) vector and spinor correspond to bosonic and fermionic states in the space-time, respectively. For each twisted sector, the H-momentum of bosonic massless states is shown in Table 1 [11, 12]. The massless bosons in the untwisted sector U_j have the H-momenta $p^i = \delta^i_j$ $(i, j = 1 \sim 3)$. The H-momenta of massless fermions are obtained from those of the corresponding bosons by the space-time supertransformation [16]. The space-time supercharge includes the H-momentum $(-1, -1, -1)/2$ for the six-dimensional internal space.

For bosons in the non-oscillated T_k sector, right-moving parts of vertex operators are obtained in the -1-picture as

$$V_{-1} = e^{-\phi} e^{iKx} e^{i\tilde{p}H} \sigma_{f\gamma}, \tag{2.3}$$

where ϕ corresponds to the bosonized superconformal ghost [16] and e^{iKx} is the four-dimensional space-time part. Right-moving parts of vertex operators for fermionic states are written in the $-1/2$-picture as

$$V_{-1/2} = e^{-1/2\phi} e^{iKx} e^{i\tilde{p}H} \sigma_{f\gamma}. \tag{2.4}$$

Similarly we can obtain vertex operators in the untwisted sector, where we do not need twist fields. We can change the picture by the picture changing operator [16], which includes the following term:

$$e^{\phi} e^{-i\alpha^i H} \partial X_i, \tag{2.5}$$

where $\alpha^1 = (1, 0, 0)$, $\alpha^2 = (0, 1, 0)$ and $\alpha^3 = (0, 0, 1)$. Note that vertex operators corresponding to the non-oscillated states include oscillators ∂X_i if we change their pictures.

Now we study on nonrenormalizable couplings of the type $V_F V_F V_B^\ell$, where V_F and V_B are the vertex operators for fermions and bosons, respectively. The vertex operators consist of several parts. Each part provides a selection rule due to charge conservation. At first we discuss the selection rule due to the space group invariance. A coupling vanishes unless a product of space group elements corresponding to states includes the identity up to conjugacy classes. Among the space group invariance, the point group invariance provides an easy selection rule. For 3-point couplings of the Z_N orbifold models, the selection rules due to the space group are explicitly obtained in ref.[12, 13]. We can extend those results to nonrenormalizable couplings. Further a product of eigenvalues, $\prod_i \gamma_i$, should be the identity.

Next we discuss the ϕ-charge conservation. To match the background ϕ-charge, the sum of the ϕ-charges of vertex operators should be -2 [16]. Thus we have to use the picture changing operator $(\ell-1)$ times in the correlation function $< V_F V_F V_B^\ell >$. That causes change of H-momenta and appearance of oscillators ∂X_i. The H-momenta should be conserved in nonvanishing couplings. Further nonvanishing couplings should be invariant under the Z_N rotation of oscillators ∂X_i for each complex plane. Note that oscillators of each T_k sector, $\partial X_{i(k)}$, transform under the Z_N rotation in a different way from oscillators of other sectors as follows,

$$\partial X_{i(k)} \rightarrow e^{2\pi i k v^i} \partial X_{i(k)}. \tag{2.6}$$

We discuss on coupling conditions of non-oscillated twisted sectors. For a concrete example, we consider the Z_4 orbifold models. There are two twisted sectors, T_1 and T_2 for matter fields. The other twisted sector T_3 corresponds to anti-matter fields. Massless states of T_1 and T_2 have the H-momenta $\tilde{p}^i = (1,1,2)/4$ and $\tilde{p}^i = (2,2,0)/4$, respectively, as shown in Table 1. The point group invariance allows $T_1^{4\ell}$-couplings, $(T_1^2 T_2)^\ell$-couplings, $T_2^{2\ell}$-couplings and their products. Here we investigate the $T_1^{4\ell}$-coupling. The corresponding correlation function $< V_{-1/2} V_{-1/2} V_{-1}^{4\ell-2} >$ has totally the H-momentum as

$(\ell-1,\ell-1,2\ell-1)$, where -1 is the contribution due to the space-time supercharge in $V_{-1/2}$. We use the picture changing operator $(4\ell-3)$ times in order to change the total picture of the correlation function into -2. At the same time we have to make the H-momentum conserved by using (2.5). As a result, the H-momentum conserved correlation function includes oscillators of T_1 as

$$(\partial X_{1(1)})^{\ell-1}(\partial X_{2(1)})^{\ell-1}(\partial X_{3(1)})^{2\ell-1}. \tag{2.7}$$

Nonvanishing correlation functions should be invariant under the Z_4 rotation (2.6) of each two-dimensional plane. This invariance requires $\ell-1=4m$ and $2\ell-1=2n$. The latter is impossible to satisfy. Thus the $T_1^{4\ell}$-couplings are not allowed. Similarly the $T_2^{2\ell}$-couplings are forbidden.

Next we study $(T_1^2T_2)^{\ell+1}$-couplings. Corresponding correlation functions with the total -2-picture include oscillators in the H-momentum conserved form as follows

$$(\partial X_1)^\ell(\partial X_2)^\ell(\partial X_3)^\ell. \tag{2.8}$$

Note that these oscillators include two types, i.e. oscillators of T_1 and T_2 and these oscillators $\partial X_{i(k)}$ transform under the Z_4 rotation differently from each other (2.6). For the first and second planes, the invariance under the Z_4 rotation requires that ℓ should be even. In the case with $\ell=4m$, we can take the following combination of oscillators:

$$(\partial X_{1(1)})^\ell(\partial X_{2(1)})^\ell(\partial X_{3(2)})^\ell. \tag{2.9}$$

This combination is invariant under the Z_4 rotation of each two-dimensional plane (2.6). In the case with $\ell=4m+2$ and $m>0$, we can take similarly the following Z_4 invariant combination of oscillators:

$$(\partial X_{1(1)})^{\ell-2}(\partial X_{1(2)})^2(\partial X_{2(1)})^{\ell-2}(\partial X_{2(2)})^2(\partial X_{3(1)})^4(\partial X_{3(2)})^{\ell-4}. \tag{2.10}$$

For $\ell=2$ we can take the Z_4 invariant combination of oscillators as follows,

$$(\partial X_{1(2)})^2(\partial X_{2(2)})^2(\partial X_{3(1)})^2. \tag{2.11}$$

Note that this combination does not correspond to the "standard" form $< V_{-1/2}V_{-1/2}V_{-1}V_0^6 >$. If the two fermions belong to T_1, the combination (2.11) is obtained as $< V_{-1/2}V_{-1/2}V_{-1}V_{-1}V_1V_0^4 >$, where V_1 corresponds to T_2. As a result, the $(T_1^2T_2)^{m+1}$-couplings are allowed if m is even.

Similarly, we can obtain selection rules for $(T_1T_1T_2)^{\ell+1}T_1^{4m}$-couplings and $(T_1T_1T_2)^{\ell+1}T_2^{2m}$-couplings. Both types are allowed if $\ell + m$ is even. The H-momentum conservation and Z_4 invariance of oscillators require nontrivial forms of correlation functions for smaller values of ℓ and m. Note that $T_1^{4\ell}T_2^{2m}(T_1T_1T_2)^n$-couplings correspond to either of the above two types.

For the other orbifold models, we can derive allowed nonrenormalizable couplings in a similar way. The results are shown in Table 2, where PGI denotes the selection rule due to the point group invariance, i.e. $\ell+2m+3n = 6p$ for the $T_1^\ell T_2^m T_3^n$-couplings of Z_6-I, $\ell + m + 2n = 3p$ for the $T_1^{2\ell}T_2^mT_4^n$-couplings of Z_6-II and $\ell + 2m + 3n + 4p = 6q$ for the $T_1^\ell T_2^m T_3^n T_4^p$-couplings of Z_6-II. For the Z_3 orbifold models, the allowed couplings have been obtained already in ref.[2, 15]. This selection rule is not difficult to understand in terms of an R-symmetry of effective field theories. For the Z_6-I orbifold models, T_3^ℓ-couplings are forbidden. On the other hand, the Z_6-II orbifold models do not allow any couplings including only one twisted sector, i.e. T_k^ℓ-couplings $(k = 1 \sim 4)$. Further $T_2^\ell T_4^m$-couplings are forbidden. For some couplings, the H-momentum conservation and Z_4 invariance of oscillators require nontrivial forms of correlation functions which include vertex operators with higher pictures.

So far we have studied the selection rules of $< V_FV_FV_B^\ell >$ including only the non-oscillated twisted sectors. Similarly we can discuss the couplings including the untwisted sectors and the oscillated sectors. For the untwisted sector, the Z_N rotation acts as (2.6) with $k = 0$. Thus the H-momentum conservation and the Z_N invariance (2.6) do not forbid the couplings UT^ℓ if T^ℓ is allowed. The H-conservation rule is very severe for the couplings including only bosons. Every massless bosonic state has the total

internal H-momentum as $\tilde{P} = \sum_{i=1}^{3} \tilde{p}^i = 1$. Thus the correlation function $< V_{-1}V_{-1}V_0 \cdots V_0 >$ has the total internal H-momentum $\tilde{P} = 2$. Therefore any couplings of only the bosons are not allowed. This rule implies that we cannot generate the μ-term through this type of the couplings and we might need a fermion condensation [6] to generate the μ-term by the nonrenormalizable couplings.

In this section we have discussed on the selection rules due to the H-momentum conservation and the Z_N invariance of oscillators. A further nontrivial selection rule is required if all twisted states sit at the same fixed point as shown in ref.[15].

3. Phenomenological implications of nonrenormalizable couplings

In this section we discuss on phenomenological implications of the results obtained in the previous section. At first we study whether the selection rules of nonrenormalizable couplings can be understood by a simple symmetry of effective field theories. We take the selection rule for the nonrenormalizable couplings in the Z_4 orbifold models as an example. We consider a Z_4 discrete R-symmetry and assign the R-charge $k/2$ to the T_k sector. In addition we assign the R-charge 1 to the fermionic coordinate of the superspace. We impose that the superpotential should have the R-charge 2 mod 4. This R-symmetry requires that ℓ should be even for the $(T_1^2T_2)^{\ell+1}$-coupling. However, there is no reason to forbid the $T_1^{8\ell+4}$-couplings and the $T_2^{4\ell+2}$-couplings.

After symmetries are broken like the anomalous U(1) symmetry breaking [1, 2, 3] and fields develop vacuum expectation values (VEVs) v, some nonrenormalizable couplings work as renormalizable coupling terms including a suppression factor $\varepsilon = v/M$, where M is the Planck scale or the string scale. Several types of nonrenormalizable couplings in Table 2 lead to strongly

suppressed couplings. That fact is favorable to the large hierarchy like the μ-term.

In ref.[17], left-right symmetric mass matrices with texture zeros are discussed to derive the quark masses and mixing angles consistent with the experimental results. Five solutions with five texture zeros are obtained. In ref.[5] the possibility of deriving such mass matrices is discussed within the framework of the extension of the standard model by a U(1) symmetry. This simple extension can lead to Solutions 1, 2 and 4 of ref.[17].

The selection rules obtained in the previous section could lead to other types of mass matrices. Here we discuss two examples corresponding to Solutions 3 and 5 of ref.[17] for the up-sector quarks.

Now we consider the Z_4 orbifold model where all of the quarks belong to T_1 and the Higgs field corresponds to T_2. Suppose that a state of T_2 sector develops the VEV. We can choose the space group elements of the states so that the (1,2), (2,1), (2,3) and (3,2) elements are forbidden. For example we assign all of the first and third generations of the quarks to the same fixed point, which is different from the fixed point corresponding to the second generation. Then we have the mass matrix proportional to the following matrix:

$$\begin{pmatrix} m_{11} & 0 & \varepsilon^4 \\ 0 & \varepsilon^4 & 0 \\ \varepsilon^4 & 0 & 1 \end{pmatrix}. \tag{3.1}$$

The (3,3) element corresponds to the renormalizable $T_1^2 T_2$-coupling. The nonvanishing elements with ε^4 are obtained through the $T_1^2 T_2^5$-coupling. If m_{11} is enough suppressed, this matrix corresponds to Solution 3. However, we have $m_{11} = \varepsilon^4$ if we take into account only the selection rules due to the H-momentum conservation, the Z_4 invariance of oscillators and the space group invariance. Following ref.[5], we can suppress this element if we assume the extra U(1) symmetry to allow the (3,3), (1,3), (2,2) and (3,1) elements as (3.1). Suppose that the i-th generation of the left-handed and right-handed

quarks have the U(1) charges α_i and α'_i, respectively. They should satisfy $\alpha_h = -\alpha_3 - \alpha'_3$ and $\alpha_3 - \alpha_1 = \alpha'_3 - \alpha'_1 = 4\beta$, where α_h and β denote the U(1) charges of the Higgs field and the field with the VEV, respectively. Then the U(1) charges of the first generation satisfy $\alpha_1 + \alpha'_1 + \alpha_h = -8\beta$. Using this extra U(1) symmetry, we obtain $m_{11} = \varepsilon^8$, which is derived from the $T_1^2 T_2^9$-coupling.

Next we consider the Z_6-I orbifold model where the Higgs field corresponds to T_2. We assign the second and third generations of the quarks to T_1 and T_2, respectively. In addition the first generation of the quarks are assigned to T_3 states with the θ-eigenvalue $\gamma = \exp[2\pi i/3]$. Suppose that a state of T_1 and a state of T_3 with $\gamma = \exp[2\pi i/3]$ develop VEVs. Then we obtain the mass matrix proportional to the following matrix:

$$\begin{pmatrix} \varepsilon^9 & \varepsilon^9 & \varepsilon^2 \\ \varepsilon^9 & \varepsilon^2 & \varepsilon \\ \varepsilon^2 & \varepsilon & 1 \end{pmatrix} . \tag{3.2}$$

The (3,3) element corresponds to the renormalizable T_2^3-coupling. The (2,3) and (3,2) elements of this matrix are induced by the $T_1^2 T_2^2$-coupling. The (2,2) element is obtained through the $T_1^4 T_2$-coupling. The (1,3) and (3,1) elements are derived from the $T_1 T_2 T_3^3$-coupling. Note that nonvanishing nonrenormalizable couplings should include $T_3^{3\ell}$ ($\ell \geq 0$) because each T_3 state is assumed to have $\gamma = \exp[2\pi i/3]$. The other elements correspond to the $T_1^3 T_3^9$-coupling or the $T_1^9 T_3^3$-coupling. This mass matrix corresponds to Solution 5 in the approximation that ε^9 is neglected. In this case we do not need an extra symmetry to derive this form. We can choose the space group elements of the states so that the space group invariance forbids the (2,3) and (3,2) elements. In this case the matrix (3.2) corresponds to Solution 3.

So far we have discussed the case where the field with the VEV belongs to the non-oscillated twisted sector. We could expect that fields in the untwisted

sector or the oscillated twisted sector develop VEVs [†]. In this case the H-momentum conservation and the Z_N invariance allow nonrenormalizable couplings with lower powers, e.g. $UTTT$. These couplings are also useful to derive the realistic mass matrices.

World-sheet instantons induce another suppression factor as $\exp[-aT]$, where T is the moduli parameter and a is a constant [9, 10]. This factor lead to other types of the hierarchy in the quark masses [19].

At last we comment on the CP phase. In the ten-dimensional superstring theories, the CP is a good symmetry [20]. In ref.[21], it is shown that the CP is unbroken in the orbifold models without background anti-symmetric tensors. The presence of the anti-symmetric tensors [22] breaks the CP symmetry in a sense of the world-sheet. Actually Yukawa couplings can have complex phases. For the 3-point couplings of the Z_3 orbifold models, the selection rule due to the space group invariance is very restrictive. If we fix space group elements of the two states, the other state to couple them is unique. Then we obtain the diagonal mass matrices when we switch off nonrenormalizable couplings, i.e. $\varepsilon \to 0$. Hence we can eliminate complex phases by rephasing fields, even though the Yukawa couplings have complex phases. For the other orbifold models, Z_4 and Z_6, the selection rules are not so restrictive. However we can always eliminate complex phases in the case with $\varepsilon \to 0$. Thus the CP is unbroken in the effective field theory, although it is broken in the world-sheet. The nonrenormalizable couplings could induce the nontrivial appearance of complex phases into the mass matrices. It is important to investigate which assignment of the quarks to the space group elements results in the mass matrices with the physical CP phase or not.

[†]The appearance of oscillated states are restricted in some cases [18].

4. Conclusion

We have studied the nonrenormalizable couplings in the orbifold models and derived the nontrivial selection rules. Some nonrenormalizable couplings lead naturally to the couplings with strongly suppressed factors. We can use the selection rules of the orbifold models to derive interesting mass matrices, e.g. Solutions 3 and 5 of ref.[17] as well as Solutions 1, 2 and 4. These analyses constrain assignments of the matter fields to the untwisted and twisted sectors. That is very useful for model building. It is also important to investigate in which case the physical CP phase appears.

In this paper we have restricted ourselves to the Z_N orbifolds which are obtained by products of three two-dimensional orbifolds. We can discuss similarly for the other Z_N orbifold models and the $Z_M \times Z_N$ orbifold models [23].

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Table 1. H-momenta

For massless bosons, H-momenta of T_k sectors are listed. The elements corresponding to the four-dimensional space-time are omitted.

Orbifold	v^i	T_1	T_2	T_3	T_4
Z_3	$(1,1,-2)/3$	(1,1,1)/3			
Z_4	$(1,1,-2)/4$	(1,1,2)/3	(2,2,0)/4		
Z_6-I	$(-2,1,1)/6$	(4,1,1)/6	(2,2,2)/6	(0,3,3)/6	
Z_6-II	$(2,1,-3)/6$	(2,1,3)/6	(4,2,0)/6	(0,3,3)/6	(2,4,0)/6

Table 2. Allowed nonrenormalizable couplings

Orbifold	Coupling	Condition
Z_3	$T_1^{9\ell+3}$	
Z_4	$(T_1^2T_2)^{2\ell+1}$	
	$(T_1^2T_2)^{\ell+1}T_1^{4m}$	$\ell+m=2n$
	$(T_1^2T_2)^{\ell+1}T_2^{2m}$	$\ell+m=2n$
Z_6-I	$T_1^{36\ell+6}$	
	$T_2^{9\ell+3}$	
	$T_1^{2\ell}T_2^m$	$\ell+m=9n+3, \ell>0, m>0$
	$T_1^{3\ell}T_3^m$	$\ell+m=4n+2, \ell>0, m>0$
	$T_2^{3\ell+3}T_3^{2m}$	$m>0$
	$T_1^\ell T_2^m T_3^n$	$\ell>0, m>0, n>0$, PGI
Z_6-II	$T_1^{2\ell}T_2^m$	$\ell+m=9n+3, \ell>0, m>0$
	$T_1^{6\ell}T_3^{2m}$	$\ell+m=2n+1, \ell>0, m>0$
	$T_1^{2\ell}T_4^m$	$\ell+2m=9n+3, 2\ell+m=9p+3, \ell>0, m>0$
	$T_2^{3\ell}T_3^{2m}$	$\ell>0, m>0$
	$T_4^{3\ell}T_3^{2m}$	$\ell>0, m>0$
	$T_1^{2\ell}T_2^m T_4^n$	$\ell=2p+1, 2\ell+2m+n=3q, \ell>0, m>0, n>0$, PGI
	$T_1^\ell T_2^m T_3^n T_4^p$	$\ell>0, n>0, m$ or $p>0$, PGI